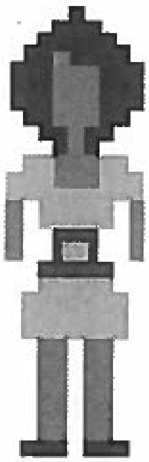

Chapter
7

Mathematics at Home

Luis, a preschooler, came to school one morning eager to tell his teacher, Ms. Luongo, about an exciting event. *"I did math last night at home!"* She asked, *"Really? What did you do?"* Luis replied, *"I helped set the table, and I had to make sure I had enough forks."*

When you think about doing math at home, what is the first thing that comes to mind? "Homework" pops up for many parents and caregivers. But as Luis discovered, opportunities to think mathematically are everywhere. Daily household activities such as folding napkins, sorting clothes, helping cook a meal, or wondering how long until bedtime can lead to meaningful mathematical conversations with your child. Mathematical thinking involves raising questions and searching for patterns—in essence, it is about figuring things out. Perhaps that night at Luis's house, there were guests for dinner. Did someone tell him how many forks to put on the table or did he figure that out himself? Everyday life presents numerous situations for problem solving.

Talking about mathematics is a worthwhile practice you can do to help your children see the connections math has to "real life." These conversations can be sparked by homework problems, grocery shopping, browsing in a toy store, or other activities in daily living. Mathematics can and should be a regular subject that you talk about with your children.

Before setting out to make math a routine part of your conversations, it is helpful to know what to expect from your children as they make sense of what they're learning in school. For example, it is important to realize that mathematical thinking does not always progress in a straight line; sometimes a child will temporarily revert back to a less sophisticated approach when learning something new. To start the discussion, we revisit some ideas about what it means for children to make sense of procedures.

Making Sense of Procedures

What does it mean for children to do meaningful mathematics? Earlier in this book, we discussed the importance of understanding not only how to use procedures but also why those procedures work. How can parents help their children make sense of the methods and procedures they're learning and using in school?

One of the crucial goals of elementary school is for children to develop mathematical methods and strategies that eventually become automatic. The procedures should make sense to children, and they should be able to explain why the procedures work. To help your children achieve this goal, you may need to take on a new role when you work and talk together. But you may not know the inner workings of these procedures, although you may easily solve the problem. It was not until recently that understanding why procedures work has been emphasized in school mathematics, so like many others, you may find some processes difficult to explain. For instance, there are few individuals who can clarify the underpinnings of the invert-and-multiply method when dividing fractions. (See page 155 to find one way to do this.)

If you did not have many opportunities to explore mathematical reasons for different procedures in your own education experience, you may find it challenging to work with your children as they do their math homework. So you and your children may need to collaborate to make sense of the mathematics they are learning.

Whether you are playing a math game or helping your child with her homework, keep these few cautionary notes in mind, and you will both be happy with the experience:

Although the ultimate goal is that children develop efficient and fluent skills, strategies, and methods, these approaches may look markedly different for each child. For instance, a young child may need to rely on counting strategies for some time before using non-counting strategies. Using non-counting strategies is one of the goals for students in the primary grades, but students also need time to develop these skills and concepts. Not all students will develop these skills and concepts at the same time in the same way. And do not assume that a child who uses counting strategies is not being efficient. Children should count objects, including their fingers, early on. As they develop new methods over time, they will no longer need to count by ones to solve problems.

Sometimes children return to simpler methods when they encounter new, more challenging ideas. Recently, a colleague reported that her first-grade students, who had demonstrated mastery of counting by fives and tens, began counting by ones when determining how many tennis balls a large bucket could hold. Reverting to that method could be related to the quantity or type of items involved. It could even be associated with the children wanting to be accurate. It is not uncommon for children to temporarily use less efficient strategies when exploring new ideas. But you'll be fascinated to see how quickly they develop efficient methods as they become comfortable with those new ideas.

You should support the notion that mathematics makes sense. Your children may use certain procedures, but do not know why they work. You can ask them to talk about the problem(s). You might work together to understand why the procedures work. There are few content areas that are more logically structured than mathematics. Encourage your children to figure out the meaning of the mathematics they are learning. If it seems that your children's math instruction in school is focused on memorizing formulas without enough opportunities to make sense of the mathematics, you may need to make it a point to talk with them about their work.

Talking About Mathematics

Children learn a great deal by watching adult behavior, so it's important that your child observe you reasoning mathematically in everyday situations. It's simple to do what's called a "think aloud." Our example is a parent thinking aloud while shopping for cupcakes for a party:

> *"Wait a minute. I need to get cupcakes for twenty-seven kids, and these cupcakes come in packs of twelve." The parent then places the packs in the cart, one by one. "So that's twelve ... um ... twenty-four ... not enough ... I need three more cupcakes ...oh yeah, there are at least three more cupcakes in this pack. Okay. That's enough, and there are some left over too."*

Depending on her age and readiness, the daughter might join in the sense making. What is important is that her parent is being a good role model for mathematical thinking. The parent is not embarrassed or nervous, and takes the time to figure out the right number of cupcake packs to buy. Subtle displays like this bring math "home" and help safeguard against math anxiety or math avoidance in children. The key is to make the invisible math you do every day visible to your child.

Talking about Mathematics with Your Child

The counterpoint to your display of everyday math is your child's conversation about the mathematics he or she is learning in school. Just as you might discuss a book that your child is reading, encourage him or her to talk to you about math class. If you show interest in the mathematics, your child may follow your lead and show more interest in it, too.

While your child may describe activities that do not sound like those in the math classroom of your childhood as you talk about math, you may discover something you never realized before, and the joy and playfulness of mathematics can become a shared experience. For example, one mother commented that as she and her third grader talked about arrays and drew pictures, she finally realized why the numbers 4, 9, 16, 25, and so on are called square numbers. Figure 7.1 on the following page represents 4 and 9 with arrays and reveals why they are called square numbers—the arrays are square shaped.

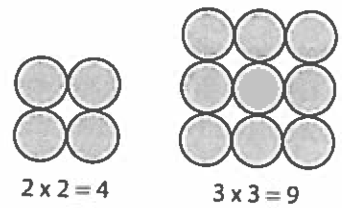

Fig. 7.1. The numbers 4 and 9 represented with square arrays

The mother said, "It had never dawned on me before, but they actually make squares. I also never realized that they increase the way they do, +5, +7, +9, and on and on. My child showed me why that was true too." How did her child do this? One possibility is by drawing nested square arrays (fig. 7.2).

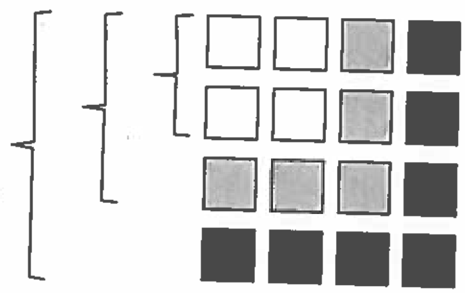

Fig. 7.2. Nested square arrays

The 2 x 2 array, made with white squares, is nested within the 3 x 3 array and the 3 x 3 array is nested within the 4 x 4 array. Notice how the number of lilac squares (5) illustrates the difference between 9 and 4, and the number of purple squares (7) illustrates the difference between 16 and 9; that is, 4 + 5 = 9 and 9 + 7 = 16. Using this diagram, a child might explain that to get the next square number, he must add squares around two edges of the 4 x 4 array. He determines that he must add four to each of the two edges plus one more for the corner, adding a total of nine squares. Therefore 16 + 9 = 25 represents the next square number. This pattern continues; to generate the square number after 25, you must add the next odd number, which is 11.

* **DO YOU KNOW** What Figurate Numbers Are?

Square numbers are one example of what mathematicians call *figurate numbers*—numbers that can be represented as a geometric shape. In addition to square numbers, there are triangular, pentagonal, and hexagonal numbers as well as other figurate numbers. There are even numbers associated with three-dimensional shapes like cubic and pyramidal numbers. The triangular numbers are frequently a topic of study in elementary school that carries forward into the study of algebra and functions. The first four triangular numbers are illustrated in figure 7.3.

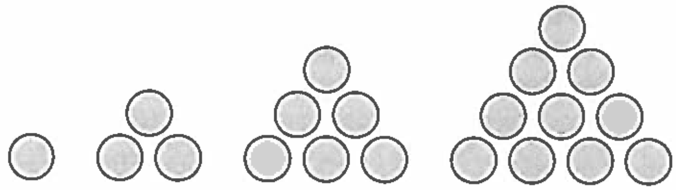

Fig. 7.3. The first four triangular numbers are 1, 3, 6, 10.

Triangular numbers are generated by adding consecutive natural numbers; that is, the first triangular number is 1; to find the next, you add 1 + 2. To find the third triangular number, you add 1 + 2 + 3, and so on. Another way of generating triangular numbers is to add another row of dots along the bottom edge to form a new larger triangle. That new row of dots will have exactly one more dot than the previous row. So, to find the next triangular number using triangles, a row of 5 more dots is added to the 10 for 15, or you could find the sum of 1 + 2 + 3 + 4 + 5. To see how to use algebra to represent these numbers, read the discussion about the staircase problem in chapter 8, page 184.

Let's look at the square numbers. The list of square numbers begins with 0, 1, 4, 9, 16, 25, 36, and continues forever. (In fact, any set of figurate numbers continues forever.) In algebra, these square numbers are referred to as *perfect squares*. Each perfect square can be represented by writing a whole number multiplied by itself or by writing the number with an exponent of two as follows:

$0^2 = 0 \times 0 = 0$; $1^2 = 1 \times 1 = 1$; $2^2 = 2 \times 2 = 4$

It may seem impossible to have a 0 x 0 array, but zero multiplied by itself is zero, and so it fits the pattern of a square number.

Following the logic of the patterns for figurate numbers reminds us how mathematics' consistent structure can be used to make sense of new mathematical situations and concepts.

When you engage in conversations about math with your child, keep these rules of thumb in mind:

- Let your child drive the conversation.
- Ask your child to explain his or her thinking with words, pictures or diagrams, and numbers.
- Remember that your child's strategy may be different from your own.
- Be patient. Explaining ideas can take time.
- Expect the unexpected. Math is a strong web of deeply interconnected ideas. The conversation could expand your child's understanding—and your own.

The vignette about buying cupcakes at the supermarket is an example of a parent being a good role model for mathematical reasoning. The parent is relaxed and demonstrates that making sense of problems is an everyday phenomenon. But what if you are a parent who is highly anxious about math? Children are remarkably perceptive and able to intuit others' feelings about matters—including mathematics—sometimes merely from body language. You may be all too aware of this and worried that you're sending the wrong messages to your children about math. There are techniques and activities you can use to de-escalate your anxiety and reduce the chances that your children pick up on it (see "Taking a Closer Look at Math Anxiety" below). Remember, we are *all* mathematical thinkers whether we are working out if we will be late for an appointment or if we have enough cash to purchase an item.

TAKING A CLOSER LOOK at Math Anxiety

Some people experience tension and worry when doing math. Math anxiety is "a feeling of intense frustration or helplessness about one's ability to do math" (Smith and Smith 1997, p. 1). It can negatively impact many facets of life including education, personal finance, and career opportunities. Even though they may not realize it, adults with math anxiety may be passing on negative feelings about math to children.

Researchers B. Sidney Smith and Wendy Hageman Smith (1997) have suggested that "math myths," such as the ones listed below, may contribute to the feeling of many people that they are simply not good at math:

MATH MYTH 1: Math requires logic, not intuition.

MATH MYTH 2: Mathematicians do problems quickly in their heads.

MATH MYTH 3: Men are better at doing math than women.

MATH MYTH 4: Mathematicians don't make computational errors.

MATH MYTH 5: There is only one correct answer.

MATH MYTH 6: Math is not a creative endeavor.

MATH MYTH 7: Some people have a math brain and others do not.

Do any of these statements sound familiar? If so, can you remember how old you were when you started to deduce these messages? Perhaps you can think of a few more. It may surprise you to know that only in the United States do people generally believe that learning mathematics depends on special ability (Mathematical Sciences Education Board 1989). It's no wonder that so many of us grew up thinking, "I'm not a math person."

If you do have math anxiety, what can you do? Our first recommendation is to slow down. When you are doing math—either in real-life situations or helping a child with homework—*take your time.* You may have painful memories of doing timed fact drills, but the truth is when you are at the store estimating how much something will cost, no one is timing you! And it's a great opportunity to model sense making for children. You may also find that relaxation techniques such as deep breathing really help. Slowly take in a breath of air, fill your lungs, hold your breath for a few seconds, and then slowly let your breath out.

If you're concerned that your own math anxiety may be "catching," here are some tips for preventing math anxiety in children:

- Be aware of messages you may be sending about math. This may be in the form of verbal or nonverbal communication.
- Allow children to use all types of reasoning to make sense of math.
- Maintain the expectation that everybody can do math, even you.

Homework

Debate continues about the role and usefulness of homework. In some countries, very little homework is assigned, and in others, homework is a cultural norm. Opinions vary from family to family, school to school, and teacher to teacher. However, in our experience, homework is a crucial issue for many parents.

Homework can serve many purposes, depending on the teacher's goals. At times, a homework assignment is given to provide students with additional practice. Other times, a teacher may want students to go home and write up an explanation for an observation that was made in class. In general, many teachers think of homework as an opportunity for students to stay engaged with the important ideas currently being addressed in class. And that comes in many forms.

One benefit of homework is that it gives parents and caregivers information about the math concepts students are exploring. Whatever the homework expectation is in your child's math classroom, it is important to understand that an activity focusing on practicing a specific skill may provide some benefit, but repeating a procedure *incorrectly* many times can be detrimental in the long run.

Where homework is completed determines, in part, how easy it is for you to monitor your child's progress. Ideally, homework is completed in a place where the parent can easily check in and be available when questions arise. Being in the same room is helpful just so you can have a sense of how your child is handling the task. Watching your child's body language can give this away. Making yourself visible, and more important, your child visible to you is crucial to knowing how things are going.

Homework can also be an opportunity to have a good math conversation. Remember to first listen to your child's reasoning and then encourage him with good questioning. The goal is not to do the assignment for him, but to help him figure it out for himself.

Good questions are those that guide children's mathematical thinking without giving away the solution to a problem. Teachers are masterful at this skill. With practice, you can learn how to do the same as you work with your children. And if you need assistance, you can always talk with your child's classroom teacher. Most teachers are eager to connect with parents and offer assistance and suggestions about the best way to help your child. This said, it can be very difficult to ask questions that encourage your child to try a different approach, consider other possible approaches, or revise an answer.

TAKING A CLOSER LOOK at Asking Questions

The primary "rule" of asking "good" questions is to pose more open-ended questions than questions with single-word answers. For example, asking "How do you know?" is likely to generate a response that reveals more about your child's understanding than asking "Is that true?" The questions you ask may also depend somewhat on the stage your child is in the problem-solving process. For example, if your child gets stuck when starting a problem, then you may want to ask him to reread the problem aloud. Sometimes children get stuck with a calculation or procedure while at other times they find errors with the solution they came up with. Because of this, we have organized a sample list of questions you can ask at critical junctures in the problem-solving process (see also https://connectedmath.msu.edu/families/homework-support/).

When your child is getting started, these are good questions to ask:

- Can you read the problem to me?
- What do you need to figure out?
- What do you already know?
- Have you solved a problem that was similar to this one?
- What parts make sense to you?
- What parts do not make sense to you?

If your child gets stuck while working on a problem, these are helpful questions to ask:

- Do you see any patterns or relationships?
- Can you try drawing a picture to make sense of the problem?
- What do you mean by . . .?
- What would happen if . . .?
- How do you know?
- Can you explain it in a different way?
- What other math does this remind you of?

After your child has found a solution, these questions are useful:

- Have you answered the question?
- Is your solution reasonable? How do you know?
- What worked?
- What did not work?

When Homework Is a Struggle

If your child is struggling with a homework assignment and has spent a reasonable amount of time and effort on it, it might be a good time to stop. Make sure you communicate with the teacher about what happened and give as much information as possible. By third grade, you might encourage your child to take on this responsibility by coaching him on what to say and what to write about the difficulties he encountered. It is important to communicate exactly what mathematical struggles your child had. Be specific and focus the communication on the mathematical ideas. It is also reasonable to expect a fifth grader to tell her teacher that she couldn't do the homework and explain why. She might write on her homework paper something like, "I got stuck when dividing these two decimal numbers because I didn't know how to divide a smaller number by something bigger with decimals." Communication with their teachers should continue as students move on to middle school, too. We know one eighth-grade math teacher who asks his students to write down how much time they spent on a problem and to note exactly "where they were in their thinking." Eventually students can take responsibility for and monitor their own learning. But no matter who communicates with the teacher about homework difficulties, always keep in mind that teachers want and need this type of information to support students as best as possible.

What If You Just Can't Help?

Even with an abundance of online tutorials and help forums, there may come a time when you feel you cannot help your child with her math homework. This is an opportune time to contact the teacher and ask what other resources or help are available. Remember, even frustrating homework experiences keep parents and teachers aware of a student's progress. Continue to talk about math with your child, ask him to show you what he is learning, find out what he likes best about his math class, and so on.

Creating Other Opportunities to Talk About Mathematics

Being a good role model for mathematical thinking and supporting your child's homework routine are not the only avenues to conversations about math. There are other activities, all with an element of fun, that provide an opening for math talk.

* **DO YOU KNOW** How to Choose Math Activity Books?

Math activity books for children are sold in many places—at bookstores, school fairs, supermarkets, toy stores, and so on. Be cautious when choosing these books. Unfortunately, many that are available can be used in ways that contradict the message of this book—that mathematics is about making sense of problems and figuring things out. In general, we recommend books that engage children in solving puzzles, whether they are number puzzles or word puzzles. Books that offer pages and pages of practice with a particular operation can be helpful if used appropriately. See "The Role of Practice" section in chapter 2 (page 40) for more insight on this issue. But such books can be detrimental if a child is not ready for such work. A far better approach is giving your children a blank piece of paper and having them make up their own problems to solve. You will learn a lot about where your children are in their thinking, both what they are comfortable with and how they might be challenging themselves to move ahead.

Games

When adults describe the math classroom of their childhoods, some may not recall *ever* playing games in school. But modern educators recognize the importance of playing math games. Games are fun and very motivating, and they have many benefits for children. Games—

- reinforce mathematical objectives;
- are repeatable and sustain interest and engagement;
- can be open-ended, allowing for multiple approaches;
- increase curiosity and motivation;
- reduce anxiety about math;
- build strategy and reasoning skills;
- lead students to talk about mathematics; and
- compel players to work mentally. (Adama Britt 2014, p. 7)

If your child's teacher sends a game home, find time to play it with him. If the game is too easy or too difficult for your child, you may think of ways to adapt the game or to adjust the difficulty level. For example, the game "Close to 100" can be made more challenging by changing the game to "Close to 1,000" (mentioned in chapter 3) or by using wild cards (in which a player assigns a digit to the card when it is drawn). For students who are ready to work with decimals, try playing "Close to 1" instead. Children will also come up with their own ways of increasing a game's difficulty level.

Quality math games do not have to be expensive: Many age-appropriate math games may be played with a simple deck of playing cards. You may have played *Go Fish* as a child, and it is still a very popular game. There is a wonderful variation of this game called *Tens Go Fish*. Recall that with *Go Fish*, a player asks a partner for a card that matches one of the cards she is holding in her hand. Each time the child finds a match, she removes that card from her hand and makes a pair with the card that she receives from her partner. If the partner does not have the card that she asks for, the partner says, "Go fish," and she must pick up a card from the deck. As play continues, it becomes the partner's turn and he asks for cards to make matches with cards that he is holding. The game ends when players can no longer make pairs.

Tens Go Fish is played using similar rules. Two simple changes in the game make this an excellent game for practicing addition facts for ten (0 + 10, 1 + 9, 2 + 8, 3 + 7, 4 + 6, and 5 + 5). First, the game is played with only forty-four instead of fifty-two cards—only the numbers 0 through 10 are played (aces can stand for one and some other face card for zero). Second, matches are made when players make pairs that sum to ten.

This game is a very popular game with first and second graders. As they play, they have many opportunities to make sums of ten. And they have chances to practice different methods for finding sums if they do not know these basic facts. For instance, a child might have a 6 card in her hand. She does not know the number she needs to make 10, but she can use a counting-on strategy to determine that she must ask her partner for a 4 card. Later in the game, if she has another 6 in her hand, she may not need to use this counting strategy—she knows that she needs a 4 card. If she does not remember, then she has another occasion to figure out that she must ask her partner for a 4 card. By its very nature, the game provides players repeated openings to figure out the card needed to make a sum of ten. Moreover, as children play the game, they can write number sentences to record the pairs they have made. This is an important way to practice writing the different facts for ten. The players can also discuss which facts they are missing on their papers and talk about how they know they have all the different ways to make ten. As you see, this is a multilayered game that helps children think about some complex ideas surrounding the facts for ten; for example, 6 + 4 = 10 and 4 + 6 = 10. Knowing these facts becomes very important as children develop chunking strategies to make sense of problems (see page 49).

There are many other card games that can be found on the Internet that are appropriate for reinforcing elementary math skills. We list a few more card games in table 7.1. You may come across these games—or very similar ones—when searching for games online. If you do not have a deck of playing cards, you can also make number cards to play these games.

Table 7.1. Card games using a deck of playing cards

.	Brief Description
Compare and **Double Compare** Grades 1–3	**The Game:** To play "Compare," divide the playing deck into two equal piles. The players turn a card faceup and determine which card has the larger number. The player with the larger number wins the cards. "Double Compare" is played the same way, but each player places two cards faceup and the sums are compared. **The Math:** Children think about the relative size of numbers, solve addition problems, and practice basic addition facts.
Salute Addition Grades 1–3	**The Game:** This game requires three players, one "adder" and two "saluters." Saluters each take one card from the deck and hold the card, face out, on their forehead so that they can only read the card of the other saluter. The adder, who can see both cards, announces the sum of the cards. The first saluter to determine what his or her card is wins the round. Players rotate positions. **The Math:** Children use part-whole reasoning, think about the relationship between addition and subtraction, and practice basic addition and subtraction facts.
Salute Multiplication Grades 3–5	**The Game:** This game is played just like the addition version described above, only there is a "multiplier" instead of an "adder." That player announces the product of the numbers of the two cards held by the saluters. The first saluter to determine what his or her card is wins the round. Players rotate positions. **The Math:** Children identify the missing factor in a multiplication statement, think about the relationship between multiplication and division, and practice basic multiplication and division facts.
Make 100 Grades 3–4	**The Game:** Players are dealt six cards. Each player chooses four cards to create 2 two-digit numbers. The goal is to make two numbers whose sum is as close to 100 as possible. Aces stand for 1, queens stand for 0, and kings and jacks are wild cards. Draw four new cards each round. Score each round by finding the difference between 100 and each player's sum. Total scores from five rounds to determine the winner. Low score wins. **The Math:** Children think about addition of two-digit numbers, addition and subtraction strategies for making 100, and place value.
Make the Most of It Grades 4–5	**The Game:** Remove kings and jacks. Aces stand for 1 and queens stand for 0. Players draw one card at a time for a total of five cards. Each card is placed in the ones, tens, hundreds, thousands, and ten thousands positions and cannot be moved. Players draw a sixth card and replace one of the five cards. High number wins the round. **The Math:** Children practice reading numbers and think about place value.

While there are some excellent commercial games available, the difficulty is in knowing which ones to choose. We discuss some of our favorites below.

24® Game (grade 4 and up). This game helps both children *and* adults develop fluency with numbers and operations. Players use the four numbers on the card in play exactly once to obtain the number 24. Players can use any combination of the four basic operations during play. The order in which the numbers are used in the operations does not matter. The game can be played independently or with up to four players. Some elementary schools host official 24 Game tournaments that are open to students across the region. For more information about this game, including rules for tournament play, see More4U and the resource list at the end of this chapter.

Let's consider the thought process a child might go through when faced with a particular card; the card in figure 7.4 has the numbers 3, 4, 11, and 13 and is similar to one from the 24 Game. Notice that the numbers are oriented around the center square. This is important to know when distinguishing a 6 from a 9 on a card!

In what ways can the child combine these numbers using any of the four operations to get a total of 24? First, she might immediately notice that she can multiply 4 by 3 to get 12. Then she may think that if she could only add 12 or multiply by 2, she would get to 24. Studying the numbers, she realizes that 13 – 11 is 2—the very number she needs to make 24. Now she can multiply the 12 she got from 4 x 3 by 2 and get 24. Symbolically, her solution can be written as $(4 \times 3) \times (13 - 11)$.

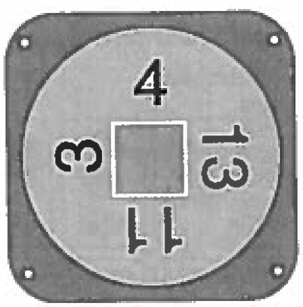

Fig. 7.4. A card similar to a 24 Game card

The cards are marked according to level of difficulty. The card in figure 7.4 has a single white dot in each corner. A single dot indicates the easiest level of difficulty. There are two other more challenging levels, indicated by two and three dots respectively. Cards with one dot usually have several different solutions. The number of possible solutions decreases as the difficulty increases. The numbers on easier cards include more factors of 24. More difficult cards also tend to involve more subtraction. Because the card in figure 7.4 is from the "easiest" category, there should be additional solutions.

READER'S CHALLENGE

Try to find at least one more solution to the 24 Game card in figure 7.4. Your goal is to use each of the numbers 3, 4, 11, and 13 exactly once to make 24.

How did you play with the numbers? Children quickly learn that it is a good idea to simply add the numbers on the card—that sometimes works! In this case, the sum of the numbers is 31, which is too much. Perhaps you tried multiplying other pairs of numbers besides 3 and 4. A child might try multiplying 3 by 11 to get 33, and then think about how he can work backwards to make 24. Since 33 – 24 = 9, he may try to make 9 with the other numbers. This works out neatly because 13 – 4 = 9, so he has, indeed, found a solution. His solution can be written as (3 x 11) – (13 – 4). The expression (13 x 3) – (4 + 11) also works. When multiple players play the game, the one who shares a correct solution first keeps the card.

Interestingly, the 24 Game can be used to address more than fluency with numbers and operations. By comparing written solutions to a card, you and your child can have conversations about equivalence, the order of operations, and properties of operations. Just having your child write his or her solution down is an important mathematical activity. And writing the solution down in a way that accurately reflects the process involves considering the use of parentheses to indicate the order in which he or she worked. All written solutions to a particular card should be equivalent mathematical expressions that use the same four numbers.

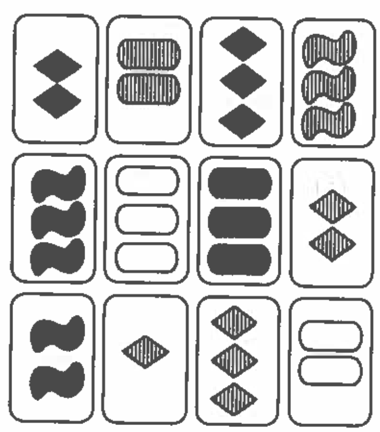

Fig. 7.5. A group of cards similar to SET cards

SET® (age six and older). The game is usually played with twelve cards lying faceup in a three-by-four array on a table (fig. 7.5). In figure 7.5 we show three of the four attributes—the shapes (squiggle, diamond, and oval), the shading (solid, stripes, and no shading), and quantity (one, two, or three shapes). Actual SET cards are also one of three colors (red, purple, or green). When younger children are learning the game, it is helpful to play with cards of only one color. Finding matches using all four attributes can be very challenging and overwhelm beginners. Children can ease into working with all four attributes as they become comfortable playing the game.

There are no turns in this game. Each player tries to identify a set, three cards that are either all the same or all different for a particular feature. If the set has the same feature, it must have that feature on all three cards in the set (for example, all three cards must have two shapes on them). If the set is of a different feature, this feature must be different for each of the cards in the set (for example, solid, stripes, and no shading).

When a player sees a set, she removes those three cards from the table. Three more cards are then laid out and play resumes. Can you find a set in figure 7.5? One possible set is one in which the shapes are different but each card has the same number of shapes and same shading (fig. 7.6).

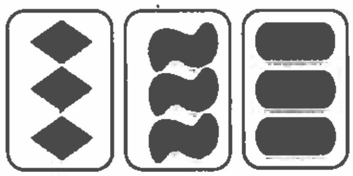

Fig. 7.6. A set with the same number of shapes, same shading, but different shapes

Generally, the more features cards share, the easier it is to see the set. When three or four features are all different, seeing the set can be a challenge. One example of this type of set is pictured below in figure 7.7. Notice that the shapes, shading, and quantities are all different in this set.

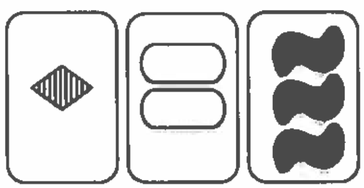

Fig. 7.7. A set in which three features are different.

Another way to use the cards is to draw two cards and then think about what type of card is needed to make a set. There are also online daily puzzles at four levels of difficulty (http://www.nytimes.com/crosswords/game/set/). The play in the daily puzzles is a little different in that some of the cards must be used in more than one set.

BLINK® (age five and older). This two-player card game is similar to SET. Players try to find a match in their stack of thirty cards to a card that is visible. To make a match, the card played must be the same color or the same shape or the same number (of shapes) as the visible card. Figure 7.8 shows three cards from the deck. To make a match with any of these cards, a player must place a card with the same color, the same shape, or the same number of shapes as one of these cards. The game ends when one of the players has played all of his thirty cards.

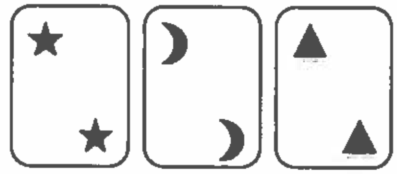

Fig. 7.8. Cards similar to those in the *BLINK* card game

This game is lots of fun for children. They love playing games that require them to be "fast," and, like SET, there are no turns. More important, they are also rapidly considering how the cards are alike or different from the three cards held in their hands. A child may first focus on the color of the cards. If none of the colors matches with one of her cards, she will switch her focus to another attribute, such as the number of shapes or the type of shape. Remarkably, a player is constantly and speedily making decisions about which card to play in which pile at the same time her partner is playing.

This game is much less taxing than SET, yet there are many similarities between the two. If children have difficulty playing SET, you might try *BLINK* instead. They will still have many of the same opportunities to compare attributes among different sets, and at the same time have a grand time playing the game.

DO YOU KNOW about After-School Events?

Schools sometimes host "Math Nights" or "Math Fairs." These are wonderful events where families learn new math games and activities to play at home. They are also great opportunities to show interest in your child's math learning, to stay connected to the school community, and, perhaps most important, to have fun learning with your child.

Computer Activities

One big difference between your schoolroom and your children's is the presence and acceptance of computers and calculators as learning tools. Some education researchers have turned their attention to how computer activities can enhance the learning experience. Researchers Julie Sarama and Douglas H. Clements (2002) found that software should relate to children's experiences and interests. They also found that it is important for children to use more open-ended computer tools as they explore math concepts. Many of the ideas we discuss here were suggested or influenced by these researchers' work.

Whether it is educational software, online games, or apps, there is a superabundance of computer activities available today. If you allow your child computer time and are looking for activities that promote understanding and fluency, keep the following recommendations in mind:

- Always try an activity yourself first before your child tries it. Are the images and reading level appropriate? Are the directions clear?
- If the resource is a drill and practice game, first determine if your child has had enough time and experience making sense of the concept or skill. When used too early in the learning process, drills may threaten a child's view of himself as a mathematical learner. Is your child ready for a drill?
- Does the game provide time and opportunity for the child to use reasoning?
- Does the game allow you to adjust the difficulty level?
- Does the game have a "free explore" component? (For example, does the software provide opportunities for the child to make designs with geometric shapes?)
- Is the game competitive or noncompetitive? Both can be helpful.
- Ask your child's teacher for recommendations, and be sure to share your success stories.

Below are a few computer activities that we like along with brief descriptions of how they might be used. (You can find the links to these activities on NCTM's website by entering the access code on the title page of this book at nctm.org/more4u.)

Meteor Multiplication (grades 3 and up). This is a good game for improving fluency with basic math facts—and children enjoy it. The premise for this game is a meteor shower. These meteors, marked with multiplication problems, are moving toward the spaceship in the middle of the screen. The spaceship is marked with the product associated with the problem on one of the meteors. The player must rotate the spaceship toward the correct meteor and blast it off the screen. Then a new number appears on the spaceship, and the player again searches to find the matching meteor as they hurtle towards the spaceship. The content range and game speed are both adjustable, making this game appropriate for grades 3 and up. There is one minute of play each round.

Factorize (grades 3 and up). Illuminations is a K–12 resource developed and maintained by the National Council of Teachers of Mathematics (http://illuminations.nctm.org). In the "Factorize" activity, children use a grid to make rectangular arrays that represent the area of a rectangle for the given number. There is a text box for the child to record all possible multiplication combinations that match the given product. For instance, if 12 is the given number, the player would make as many different rectangles with an area of 12 as she can (i.e., rectangles with dimensions 1 by 12, 2 by 6, or 4 by 3). Children can use this game to explore multiplication ideas (arrays and area) or to practice basic facts (to read more about arrays, see chapter 4, page 96). They can also investigate factors for different numbers.

READER'S CHALLENGE

Consider again the possible rectangle dimensions for a rectangle of area 12 using only whole number dimensions (1 by 12, 2 by 6, or 4 by 3). Do you see any interesting relationships among the three factor pairs? Children may notice that if the first factor in a factor pair is doubled and the second factor halved, you get the next factor pair. So 2 x 6 becomes 4 x 3, and both of these products are 12. This "doubling and halving" strategy may also be physically apparent in the dimensions of the corresponding rectangles on the screen.

Base Blocks (grades 2 and up). The National Library of Virtual Manipulatives (http://nlvm.usu.edu/en/nav/vlibrary.html) project began more than twenty years ago, and its many Java applets function as "virtual manipulatives." The Base Blocks activity may be used as a free-explore activity, or children may choose to quiz themselves by representing given numbers with the virtual base-ten blocks. The difficulty level is adjustable.

Factor Dazzle (grades 3 and up). To play games on the Calculation Nation® website (http://calculationnation.nctm.org), students must first assign themselves a screen name and password. After choosing a game, players decide if they want to challenge themselves or challenge other players from around the world. To play the game "Factor Dazzle," a player begins by selecting a number from a list of given numbers. Suppose the player chooses the number 36. Her opponent must find all the factors for 36. The player would collect 36 points; however, her opponent would collect the sum of all of the other factors of 36, so the opponent would get a score of 1 + 2 + 3 + 4 + 6 + 9 + 12 + 18 for a total of 55 points. Not only do students practice factoring and learn more characteristics of specific numbers, they improve their strategies with repeated play. What would be a good number to choose if you were selecting first from the set of whole numbers from 1 to 36 ?

Fraction Flags (grades 2 to 4). The school district of Oswego, New York, has available on its website a list of interactive games for families to explore, including two "Fraction Flag" activities that even second graders can do. Players must design a flag using the appropriate number of colors, and each color must exactly cover the correct portions of the array (the "flag"). There is no one "right" way to complete the flag. For instance, if the player designs a flag using halves, she fills half the squares using one color and the other half using a different color. It does not matter which squares are one or the other color as long as half of the squares are filled using each color.

Monster Squeeze (ages four to seven). This is an app from Everyday Mathematics® available for purchase on the publisher's website (https://www.mheonline.com). Two players take turns guessing the other player's number. Each time the player makes a guess, the monster's arms move closer and closer to the target number along a number line. This "guess my number" activity helps children become familiar with the relative size of numbers by comparing numbers and helps them to visualize the position of numbers on a number line.

Counting with the Very Hungry Caterpillar (up to five years old). This is an app for young children available for purchase on iTunes (https://itunes.apple.com). To play, the child counts pictures of different food items that the Hungry Caterpillar might eat. There are five different levels in the game. At each level, the child is prompted by a voice-over to select one or more food items to count. If the child correctly or incorrectly selects the number of items in the picture, that game continues. Once an item is selected, a hole appears where the caterpillar has eaten through the food item. At each level, the child may be asked to count the items for different situations. For level five, for instance, the child is prompted to choose the number of a particular type of food item to eat. As the child selects the number of items from a screen with several different types of foods, a voice-over counts the item(s) that the child selects. The child has repeated experiences counting small collections of food items. Apps such as these may help children build counting strategies because they reinforce one-to-one correspondence; one touch means one count.

These are just a few of the many computer resources available today. As you know, technological assets are continually being created and revamped. This can be a challenge for parents, but it is also exciting to explore new games and other programs as they become available. Get your child involved in the review process too. For example, you might ask, "What's the math?" to emphasize the importance of the mathematical aspects of the game.

DO YOU KNOW How to Use Math Games?

Games must be fun for children. If they're not, children won't want to play. Children won't like every game, so expect some trial and error as to what works for your child. Although it is important that your child be mentally stimulated, your child should enjoy these challenges and be eager to become more successful as she or he plays the game. What makes for a good game? Games should not be cumbersome. Sometimes the simplest of math games can be the most fun; think of card games such as "War" or "Compare." Certainly the game should be engaging for your child and allow your child to make different choices and decisions during play. And play competitive as well as noncompetitive games. Most important, games should offer opportunities for your child to reason about math ideas.

Final Thoughts

Even if you "hated math" as a student, you can still be a successful mathematics role model for your children: Talk to your kids about what they're learning in math class; be an attentive helpmate as they grapple with their homework; join in the fun of math games; let your child see you make sense of mathematics in everyday situations, whether it's calculating the tip for the server at a restaurant or figuring out how long to cook the turkey (maybe he or she will help you solve these problems!). As your children move through the elementary grades into middle school and then high school, your role will change, but the goal is the same—that your children be successful math learners and can make sense of mathematics whether they are working on school assignments or curious about the rock ripples in water or how birds fly. By focusing on the ingenuity of your child's mathematical thinking, you may begin to find elementary mathematics a captivating and inviting subject after all.

Throughout *It's Elementary*, we've discussed the importance of making connections between and among mathematical concepts as well as "laying foundations" and "building on skills and processes." In the final chapter, we discuss the continuing development of these connections in middle school and high school by focusing on one problem that morphs from grade to grade, starting in first grade and continuing through high school.

Things to Remember

- Children's mathematical development doesn't follow a straight line. They may revert to simpler techniques when confronted with new concepts, or they may understand one idea but not how it is related to another.
- If your child has been assigned math homework, don't do it for them. Do be supportive and offer assistance when you can.
- Let your child drive the conversation.
- Remember that your child's strategy may be different from your own.
- Be patient.
- Expect the unexpected.
- Have fun.